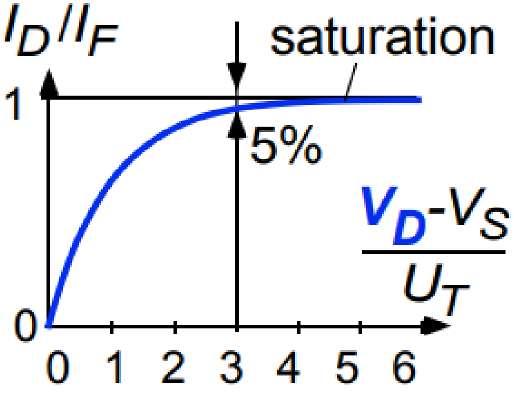
TR-18

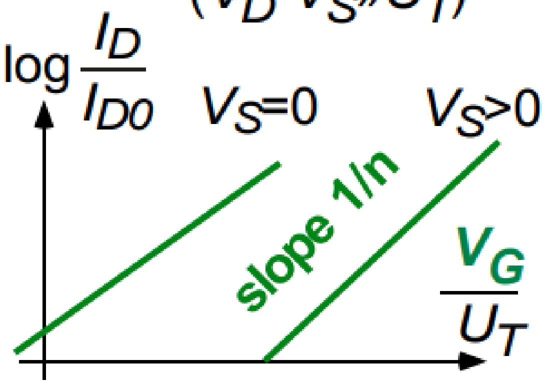
FORWARD CHARACTERISTICS IN WEAK INVERSION

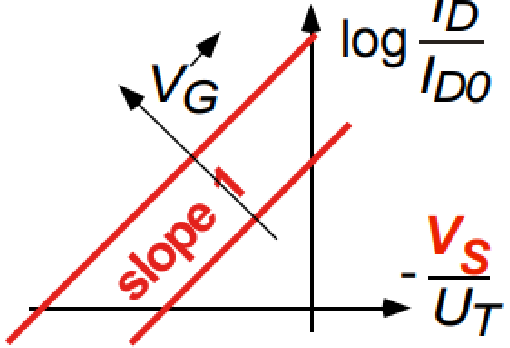
$$I_D = I_{D0}\, e^{\frac{V_G}{nU_T}} \left(e^{-\frac{V_S}{U_T}} - e^{-\frac{V_D}{U_T}} \right)$$

- where $I_{D0} = I_S\, e^{-\frac{V_{T0}}{nU_T}}$ (saturation current for $V_G=V_S=0$)

- output (V_G, V_S = const.)
- transfer from gate ($V_D-V_S \gg U_T$)
- transfer from source ($V_D-V_S \gg U_T$)







$\boxed{I_D \propto 1 - e^{-\frac{V_D-V_S}{U_T}}}$ $\boxed{I_D \propto e^{\frac{V_G}{nU_T}}}$ $\boxed{I_D \propto e^{-\frac{V_S}{U_T}}}$

- Barrier-controlled device; diffusion of "**majority**" carriers:
  - channel may be much longer than diffusion length in the substrate.

DRAIN CURRENT IN WEAK INVERSION

- $-Q_i/C_{ox} = 2nU_T e^{\frac{V_P - V}{U_T}} \ll 2nU_T$ thus: $I_{F,R} = 2n\beta U_T^2 e^{\frac{V_P - V_{S,D}}{U_T}}$
- Definition: **specific current** of the transistor: $\boxed{I_S = 2n\beta U_T^2}$
- By using the approx. $V_P = (V_G - V_{T0})/n$, this yields:

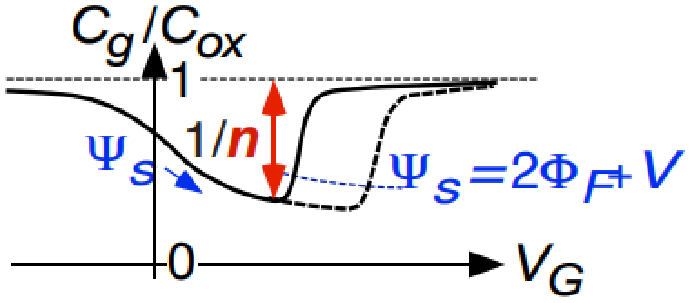
$$\boxed{I_D = I_S e^{\frac{V_G - V_{T0}}{nU_T}} \left(\underbrace{e^{-\frac{V_S}{U_T}}}_{I_F} - \underbrace{e^{-\frac{V_D}{U_T}}}_{I_R} \right)}$$

for I_F and $I_R \ll I_S$

  - exponentially dependent on V_{T0}
  - therefore, the transistor must be biased by **imposing I_D**
- Slope factor $\boldsymbol{n}$ results from capacitive divider C_{ox}/C_d (depletion capacitance)
  - it can therefore be found from the $C_g(V_G)$ curve:
  - it can be evaluated at $\Psi_s = 2\Phi_F + V_S$:

$$\boldsymbol{n} = 1 + \frac{\Gamma_b}{2\sqrt{\Psi_s}} = \boxed{1 + C_d/C_{ox} \cong 1 + \frac{\Gamma_b}{2\sqrt{2\Phi_F + V_S}}}$$

weak inversion only



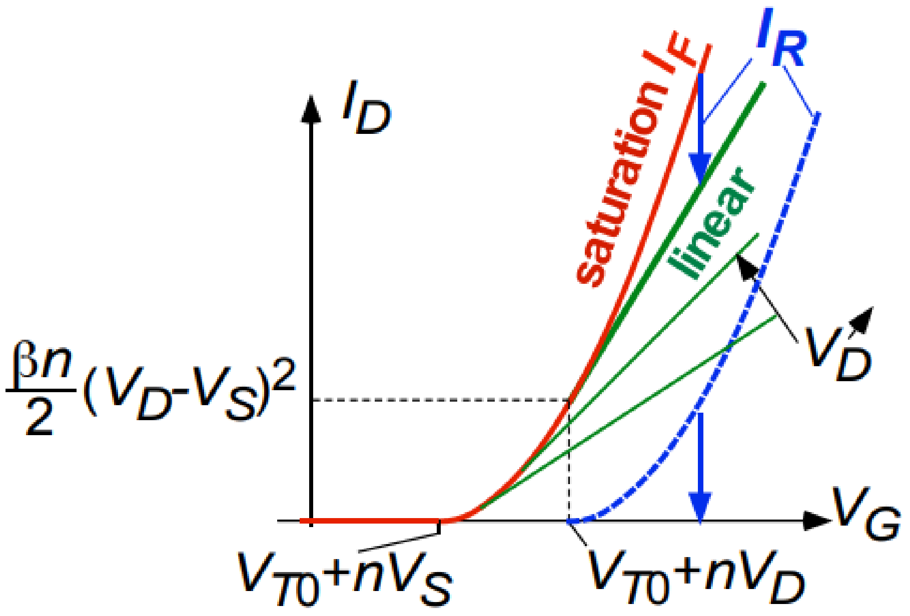
FORWARD CHARACTERISTICS IN STRONG INVERSION

Saturation: for $V_D \geq V_P$

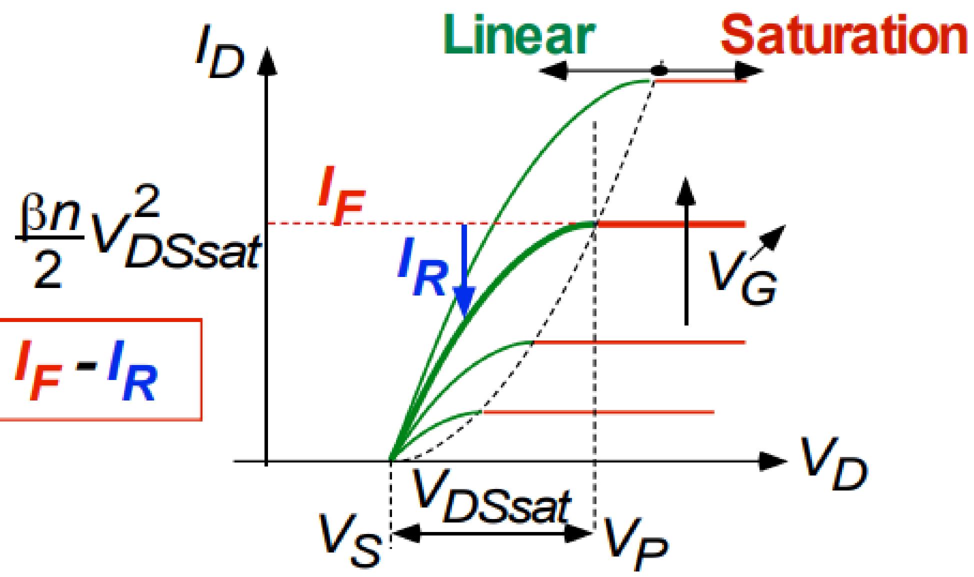
$$I_D = \frac{\beta n}{2}(V_P - V_S)^2 = \frac{\beta n}{2} V_{DSsat}^2 = \frac{\beta}{2n}(V_G - V_{T0} - nV_S)^2 = I_F$$

Linear: for $V_D \leq V_P$

$$I_D = \beta (V_D - V_S)\left[V_G - V_{T0} - \frac{n}{2}(V_D + V_S)\right] = I_F - I_R$$



$I_D = I_F - I_R$



Transfer characteristics | for V_S=const. | Output characteristics

DRAIN CURRENT IN STRONG INVERSION

- $-\frac{Q_i}{C_{ox}} = n(V_P - V)$, thus $I_{F,R} = \frac{\beta n}{2}(V_P - V_{S,D})^2$ for $V_{S,D} < V_P$
- By using the approx. $V_P = \frac{V_G - V_{T0}}{n}$ this yields, for the forward mode ($V_D \geq V_S$):

Saturation: $I_D = I_F = \frac{\beta n}{2}(\underbrace{V_P - V_S}_{V_{DSsat}})^2 = \frac{\beta}{2n}(V_G - V_{T0} - nV_S)^2$ for $V_D \geq V_P$

Linear: $I_D = I_F - I_R = \beta(V_D - V_S)[V_G - V_{T0} - \frac{n}{2}(V_D + V_S)]$ for $V_D \leq V_P$

Blocked: $I_D = 0$ for $V_S \geq V_P$

where $\beta = \mu C_{ox} W/L$

CONTINUOUS MODEL WEAK-STRONG INVERSION

$$I_{F,R} = \beta \int_{V_{S,D}}^{\infty} -\frac{Q_i}{C_{ox}}\, dV \rightarrow i_{f,r} = \frac{I_{F,R}}{I_S} = \int_{V_{S,D}}^{\infty} q_i\, dV/U_T \quad \text{where} \quad \boxed{q_i = \frac{-Q_i}{2nU_T C_{ox}}}$$

- Now: $-dV = U_T(2dq_i + dq_i/q_i)$, hence: $i_{f,r} = \int_0^{q_{s,d}} (2q_i+1)dq_i = |q_i^2 + q_i|_0^{q_{s,d}}$

  (see TR-11) (where $q_{s,d} = q_i$ at S,D)

- Thus: $i_{f,r} = \dfrac{I_{F,R}}{I_S} = (q_{s,d}^2 + q_{s,d})$

  with: $v_{s,d} = \dfrac{V_P - V_{S,D}}{U_T} = 2q_{s,d} + \ln q_{s,d}$

  parametric equations of $I_{F,R}(V_P - V_{S,D})$

- Solving the first equation for $q_{s,d}$ yields: $\boxed{q_{s,d} = \dfrac{\sqrt{1+4i_{f,r}} - 1}{2}}$
  - which, introduced in the second equation, provides $(V_P - V_{S,D})$ of $I_{F,R}$:

$$\boxed{v_p - v_{s,d} = \sqrt{1+4i_{f,r}} - 1 + \ln \frac{\sqrt{1+4i_{f,r}} - 1}{2}}$$

only **3** parameters:

$\boxed{n,\ V_{T0} \text{ and } \beta \text{ (or } I_S)}$

- This equation **cannot be inverted** to provide $I_{F,R}(V_P - V_{S,D})$

ALTERNATIVE CONTINUOUS MODEL WEAK-STRONG INVERSION

- Interpolation formula: $$\boxed{I_{F,R} = I_S \ln^2\left(1 + e^{\frac{V_P - V_{S,D}}{2U_T}}\right)}$$

  - for $V_{S,D} \ll V_P$ or $I_{F,R} \gg I_S$ $\quad I_{F,R} = \frac{\beta n}{2}(V_P - V_{S,D})^2$ strong inversion

  - for $V_{S,D} \gg V_P$ or $I_{F,R} \ll I_S$ $\quad I_{F,R} = I_S\, e^{\frac{V_P - V_{S,D}}{U_T}}$ weak inversion

- $\boxed{I_D = I_F - I_R}$ includes all field-effect modes of operation (but not the bipolar mode).

- Only **3** parameters: $\boxed{n,\ V_{T0} \text{ and } \beta \text{ (or } I_S)}$

COMPARISON OF CONTINUOUS MODELS

- Definitions: $v_p\text{-}v_{s,d} = \dfrac{V_P\text{-}V_{S,D}}{U_T}$ $\qquad i_{f,r} = \dfrac{I_{F,R}}{I_S}$

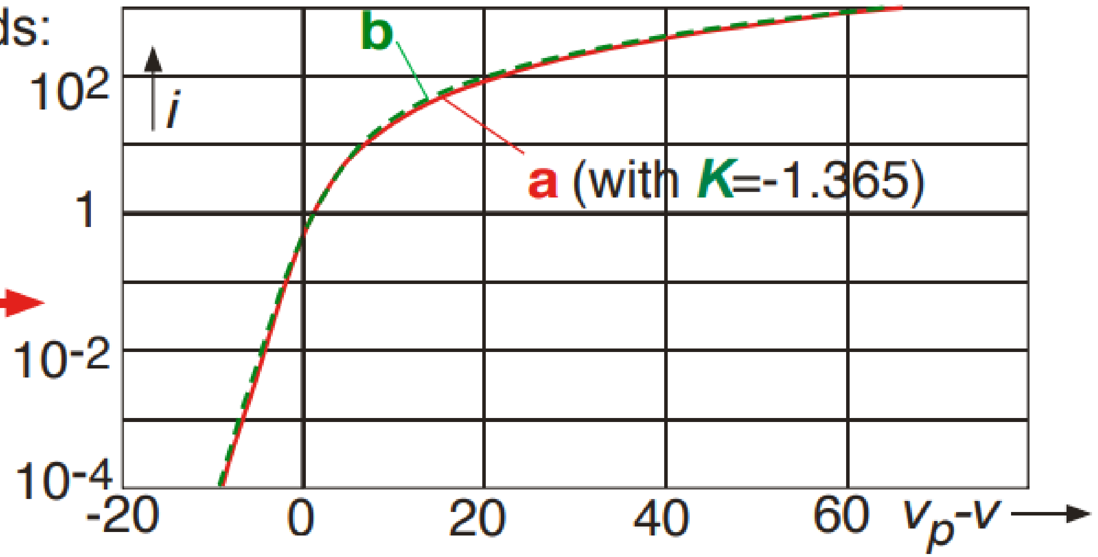
- Charge-based model (non-invertible):

$$v_p\text{-}v = \sqrt{1+4i} + \ln(\sqrt{1+4i} - 1) + K$$ curve **a** $\quad (v = v_{s,d}$ for $i = i_{f,r})$

  - constant K depends on the exact definition of V_{T0}
    - K =-(1+ln2) = -1.693 with definition in strong inversion approx.

- Simple interpolation $i = \ln^2(1+e^{(v_p\text{-}v)/2})$ (alternative continuous model)
  - can be inverted, which yields:

$$v_p\text{-}v = 2\ln(e^{\sqrt{i}} - 1)$$ curve **b**



- for K = -1.365:
  - **a** and **b** coincide at i =1
  - very similar elsewhere

INVERSION COEFFICIENT

- Inversion coefficient IC characterizes the mode of operation.

- Definition: IC = the larger of I_F/I_S or I_R/I_S
  - hence I_F/I_S in forward mode and I_R/I_S in reverse mode
  - with $I_S = 2n\beta U_T^2$ (10 to 1000 nA for $W = L$)

weak inversion: $IC \ll 1$

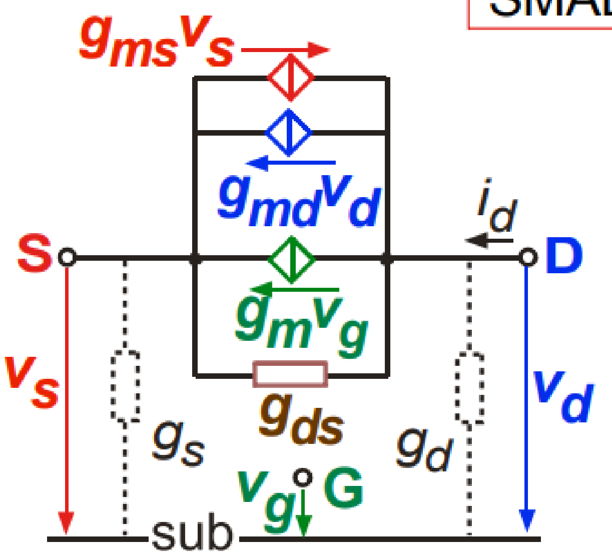
moderate inversion: IC around 1

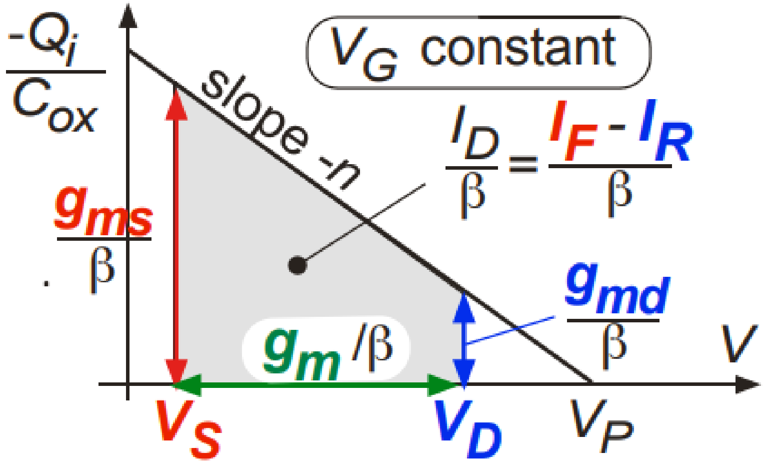
strong inversion: $IC = \left(\dfrac{V_{DSsat}}{2U_T}\right)^2 \gg 1$

E. Vittoz, “Micropower Techniques”, in *Design of VLSI Circuits for Telecommunication and Signal Processing*, Editors J.Franca and Y.Tsividis, Prentice Hall, 1994, page 62.

SMALL-SIGNAL MODEL

TR-27



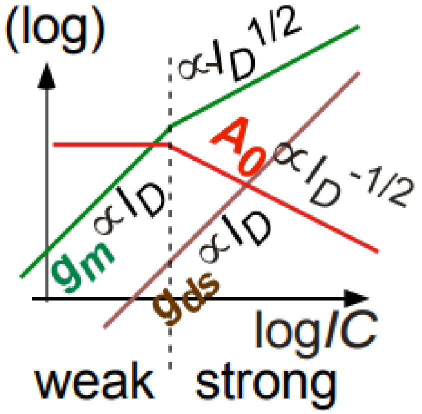


- General:
  - **S**,**D** transcond.
  - **G**ate transcond.

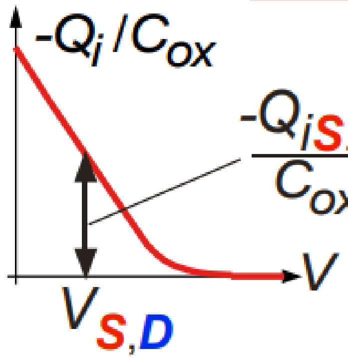
	weak	strong inversion
$g_{ms,d}$	$\frac{I_{F,R}}{U_T}$	$\beta n\,(V_P - V_{S,D}) = \frac{2I_{F,R}}{V_P - V_{S,D}} = \sqrt{2\beta n\, I_{F,R}}$
g_m	$(g_{ms} - g_{md})/n$	

- In **saturation** $I_R \ll I_F$ hence $I_D = I_F$ and $g_{md} = 0$
  - thus:
  - residual SD conductance:
  - intrinsic voltage gain:

$g_{ms} = ng_m$	$\frac{I_D}{U_T}$	$\beta n V_{DSsat} = \frac{2I_D}{V_{DSsat}} = \sqrt{2\beta n I_D}$
g_{ds}	I_D/V_M	
$A_0 = \frac{g_m}{g_{ds}}$	$\frac{V_M}{nU_T}$	$\frac{2V_M}{nV_{DSsat}} = \frac{V_M}{nU_T\sqrt{IC}}$



CONTINUOUS MODEL FOR TRANSCONDUCTANCE



$$\frac{-Q_{iS,D}}{C_{ox}} = \frac{g_{ms,d}}{\beta}, \text{ thus: } g_{ms,d} = -Q_{iS,D}\beta/C_{ox}$$

- Now: $q_{s,d} = \dfrac{-Q_{iS,D}}{2nU_TC_{ox}} \overset{\text{(see TR-17)}}{=} \dfrac{\sqrt{1+4i_{f,r}}-1}{2}$ where $i_{f,r} = I_{F,R}/I_S = \dfrac{I_{F,R}}{2n\beta U_T^2}$
  - thus:

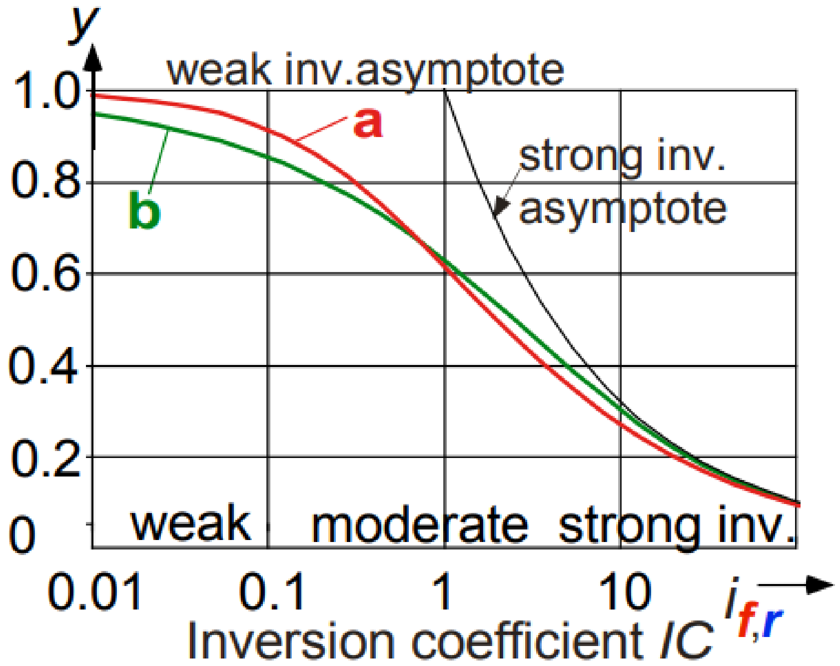
$$\boxed{y = \frac{g_{ms,d}}{I_{F,R}/U_T} = \frac{2}{1+\sqrt{1+4i_{f,r}}}}$$ curve **a**

- Alternative continuous model:
  (math. interpolation, see TR-18)

$$i_{f,r} = \ln^2\left(1 + e^{\frac{V_P - V_{S,D}}{2U_T}}\right)$$

  - yields, by differentiation:

$$\boxed{y = \frac{g_{ms,d}}{I_{F,R}/U_T} = \frac{1-e^{-\sqrt{i_{f,r}}}}{\sqrt{i_{f,r}}}}$$ curve **b**

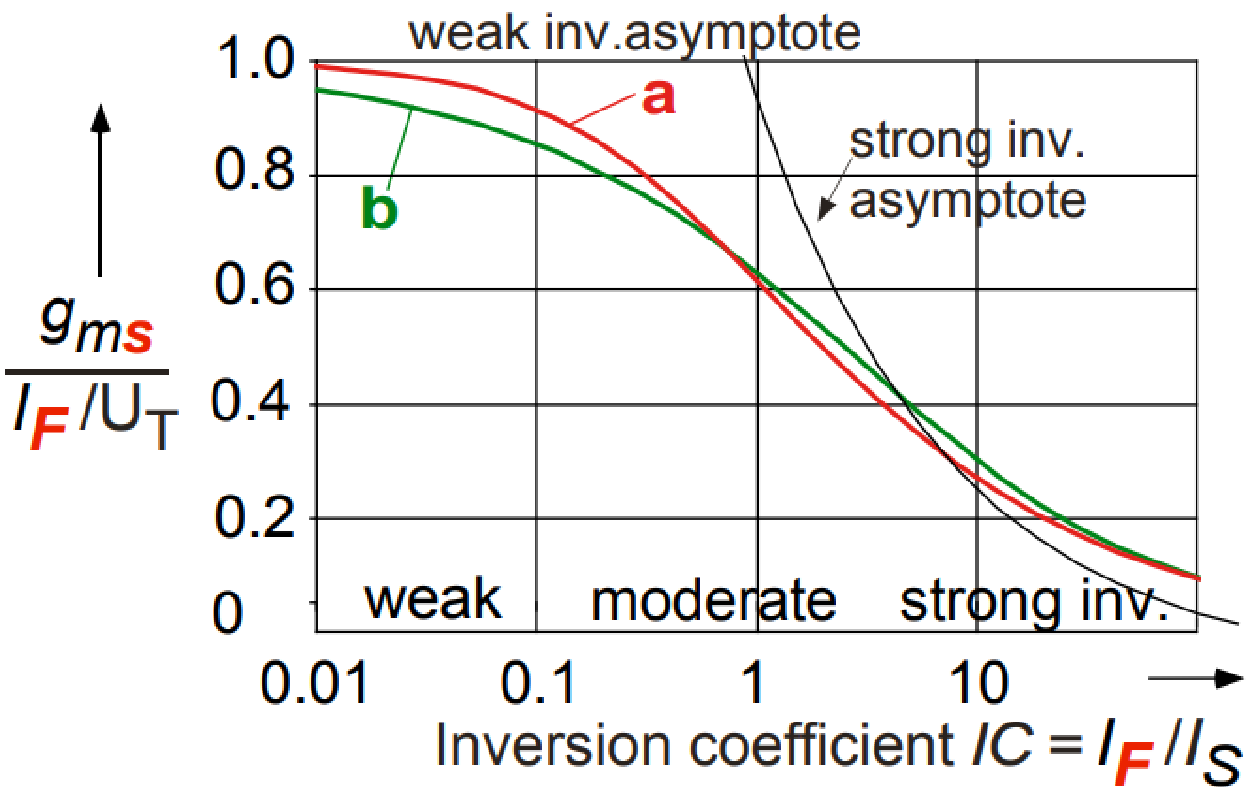


CONTINUOUS MODEL OF TRANSCONDUCTANCE

(summary)

$$g_{ms} = \frac{I_F}{U_T} \cdot \frac{2}{1+\sqrt{1+4IC}}$$ curve **a**

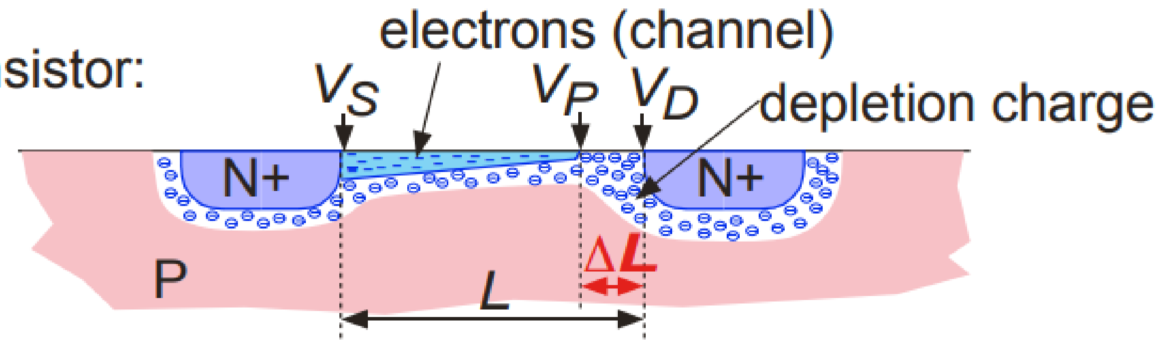
alternative model of TR-20: $$g_{ms} = \frac{I_F}{U_T} \cdot \frac{1 - e^{-\sqrt{IC}}}{\sqrt{IC}}$$ curve **b**



- Transistor sizing by the gm/ID methodology **[P.Jespers *et al*]**

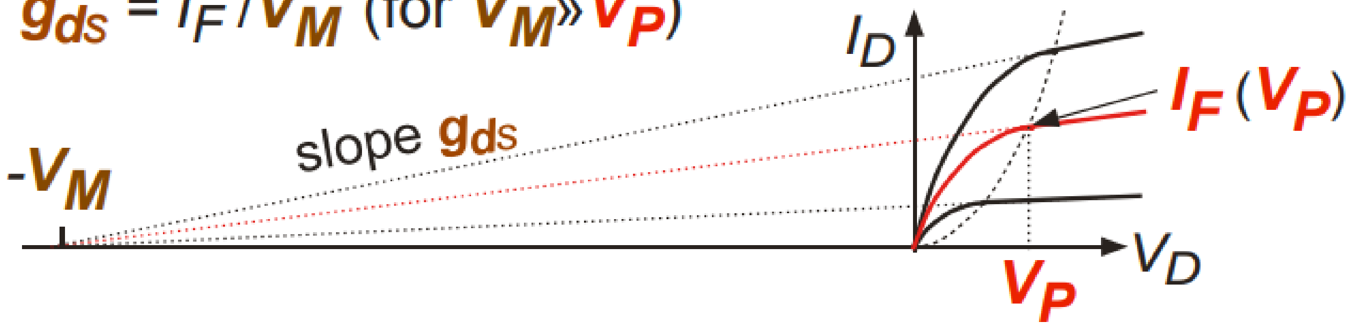
CHANNEL LENGTH MODULATION

- Forward-saturated transistor:



- Voltage drop $V_D - V_P$ requires a depletion zone of length $\Delta L = f(V_D - V_P)$
- Current I_D in saturation therefore increases slightly with V_D
- This effect can be approximated by a conductance in saturation:

$$g_{ds} = I_F / V_M \quad (\text{for } V_M \gg V_P)$$



- Channel-length modulation voltage V_M proportional to L and approximately independent of current (increases slowly with IC)

SILICON - OXIDE INTERFACE NOISE

- Due to trapping of carriers and mobility fluctuations

- Simplest possible modelling by an input noise voltage of spectral density:

$$\boxed{S_V = 4kT\frac{\rho}{fWL}}$$

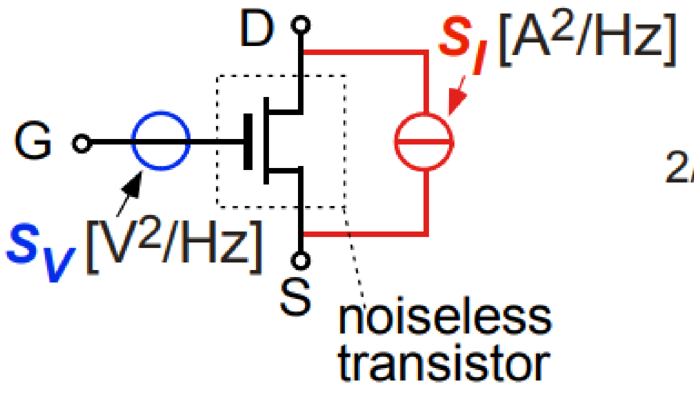
ρ $\left[\frac{Vm^2}{As}\right]$, process-dependent

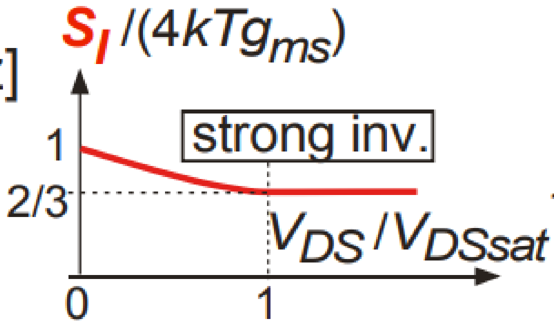
1/f flicker noise

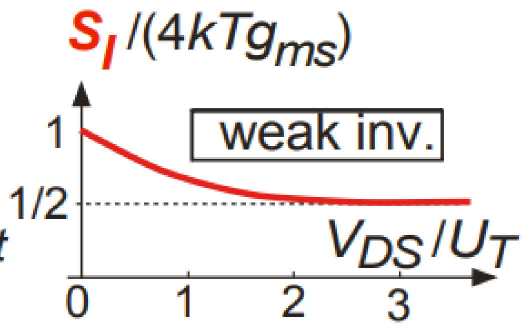
gate area

- **Process-dependent** parameter ρ:
  - practical range: 0.001 to 1 $\frac{Vm^2}{As}$
  - often smaller for P-channel transistor
  - slightly dependent on *IC*: min. around *IC*=1 (neglected here)
  - proportional to $C_{ox}^{-\alpha}$, with α = 1 to 2
    thus reduced with scaled down process

MODEL OF THE NOISY TRANSISTOR







$S_V = 4kT\dfrac{\rho}{fWL}$ (flicker noise, approximately bias-independent)

- Total equivalent noise voltage density referred to the gate input:

$$S_V + \frac{S_I}{g_m^2} \equiv 4kT R_N$$; defines equivalent **input noise resistance R_N**

- In **saturation**: $g_m = g_{ms}/n$, thus:

$$\boxed{R_N = \frac{\rho}{fWL} + \begin{cases} \dfrac{n}{2g_m} & \text{(weak)} \\[2ex] \dfrac{2n}{3g_m} & \text{(strong)} \end{cases}}$$

$R_N g_m = \gamma$ (noise excess factor)

MISMATCH OF TRANSISTORS

TR-37

- Mismatch of of pair of similar transistors T_1, T_2 characterized by:

$$\delta V_T = V_{T01} - V_{T02} \text{ with } \sigma_T = \sigma(\delta V_T)$$ (typical range: 1 to 10 mV)

$$\frac{\delta\beta}{\beta} = \frac{\beta_1 - \beta_2}{\beta} \quad \text{with } \sigma_\beta = \sigma\left(\frac{\delta\beta}{\beta}\right)$$ (typical range 0.2 to 20%)

- σ_T and σ_β depend on:
  - process
  - gate dimensions W,L ($\propto 1/\sqrt{WL}$)
  - layout
  - reduced with scale-down for same W and L.

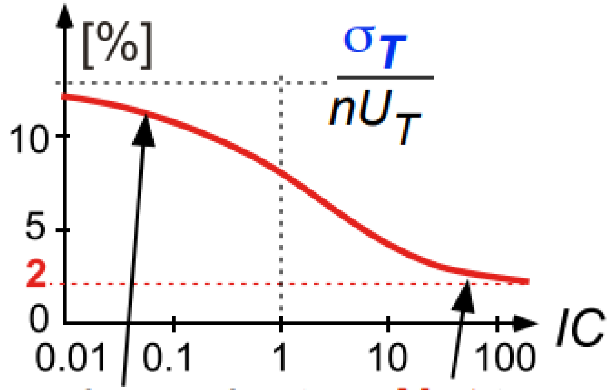
- Resulting mismatch of:
  - drain currents (same V_G)

$$\sigma\left(\frac{\delta I_D}{I_D}\right) = \sqrt{\sigma_\beta^2 + \left(\frac{g_m}{I_D}\sigma_T\right)^2}$$

(no correlation assumed)

  - gate voltages (same I_D)

$$\sigma(\delta V_G) = \sqrt{\sigma_T^2 + \left(\frac{I_D}{g_m}\sigma_\beta\right)^2}$$



Temp.dependent

Not temp.dependent

example

σ_β	= 2%
σ_T	= 5mV

nU_T=40mV